COMPLETE POSITIVITY AND CORRELATED NEUTRAL KAONS

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Abstract

In relation with experiments on correlated kaons at ϕ-factories, it is shown that the request of complete positivity is necessary in any physically consistent description of neutral kaons as open quantum systems.

Non-standard phenomenological models incorporating loss of quantum coherence and entropy increase can be studied in the broad framework of open quantum systems.[1]-[3] These systems can be modelled as being small subsystems in "weak" interaction with large environments. The reduced dynamics for the subsystem is obtained by eliminating, *i.e.* tracing over, the environment degrees of freedom; in the markovian approximation, justified in most physical contexts, it consists of a set of completely positive linear transformations, with forward in time composition law (semigroup property) and entropy increase. These maps form a so-called quantum dynamical semigroup.

This description is rather general and has been used to model a large variety of physical situations, ranging from the study of quantum statistical systems,[1]–[3] the analysis of dissipative effects in quantum optics,[4]–[6] the treatment of the interaction of a microsystem with a macrosystem.[7]–[9] It has also been recently applied to the analysis of the evolution and decay of the system of neutral kaons.[10]–[14] (For motivations and earlier results, see [15]–[22]).

According to standard phenomenology,[23] kaon states can be effectively described by 2×2 density matrices ρ acting on a two dimensional Hilbert space. The time evolution ω_t will transform the initial state ρ into the state $\rho(t) \equiv \omega_t[\rho]$, at time t. A consistent statistical description of the initial density matrix ρ as a state is assured by the positivity of its eigenvalues that are interpreted as probabilities. Clearly, for this description to hold for all times, the evolution map ω_t must be positive, *i.e.* it must preserve the positivity of the eigenvalues of $\rho(t)$, for any t.

Complete positivity[1]–[3] is a more stringent condition; it guarantees the positivity of the eigenvalues of density matrices describing states of correlated kaons, *e.g.* those produced in ϕ-meson decays. States of entangled, but not dynamically interacting kaons, evolve according to the factorized product $\omega_t \otimes \omega_t$ of the single-kaon dynamical maps.[12] If ω_t is not completely positive, there are instances of correlated states that develop negative eigenvalues; in such cases, their statistical and physical interpretation is lost.[13] The aim of this note is to show that the issue of complete positivity is not only theoretical, but can be given experimental relevance. Indeed, we shall explicitly find that some experimentally accessible kaon observables, defined to be positive, would return, in absence of complete positivity, negative mean values.

The evolution in time of any kaon state ρ can be described in general by an equation of the following form:

$$\frac{\partial \rho(t)}{\partial t} = -iH_{\text{eff}}\, \rho(t) + i\rho(t)\, H_{\text{eff}}^{\dagger} + L_D[\rho(t)] \ . \tag{1}$$

The first two pieces in the r.h.s. give the standard hamiltonian contribution, while L_D is a linear map that encodes possible dissipative, non-standard effects.

The effective hamiltonian,

$$H_{\text{eff}} = M - i\Gamma/2 \ , \tag{2}$$

includes a non-hermitian part describing the kaon decay; its generic form further encodes possible indirect CP and CPT violations. It is convenient to introduce the even and odd

CP eigenstates $|K_1\rangle$, $|K_2\rangle$ and the eigenstates of $H_{\rm eff}$

$$|K_S\rangle = \frac{|K_1\rangle + \epsilon_S|K_2\rangle}{\sqrt{1+|\epsilon_L|^2}}\ ,\ |K_L\rangle = \frac{\epsilon_L|K_1\rangle + |K_2\rangle}{\sqrt{1+|\epsilon_S|^2}}\ , \tag{3}$$

such that $H_{\rm eff}|K_S\rangle = \lambda_S|K_S\rangle$ and $H_{\rm eff}|K_L\rangle = \lambda_L|K_L\rangle$. The entries of the matrices M and $\mathit{\Gamma}$ can be expressed in terms of the two complex parameters ϵ_S, ϵ_L and the K_S, K_L masses, m_S, m_L, and decay widths γ_S, γ_L, so that $\lambda_S = m_S - i\gamma_S/2$, $\lambda_L = m_L - i\gamma_L/2$. For later use, we introduce the following positive combinations: $\Delta\Gamma = \gamma_S - \gamma_L$, $\Delta m = m_L - m_S$, as well as the complex quantities $\Gamma_\pm = \Gamma \pm i\Delta m$ and $\Delta\Gamma_\pm = \Delta\Gamma \pm 2i\Delta m$, with $\Gamma = (\gamma_S + \gamma_L)/2$.

The form of the additional piece $L_D[\rho]$ is uniquely fixed by the physical requirements that the complete time-evolution ω_t needs to satisfy; as already mentioned in the introductory remarks, these are semigroup property, entropy increase and complete positivity. Expressing the kaon state ρ as the 4-vector $(\rho_0, \rho_1, \rho_2, \rho_3)$ along the basis consisting of the identity matrix σ_0 and the Pauli matrices σ_i, $i = 1, 2, 3$, the additional linear piece L_D acts as the 4×4 matrix:[10]

$$[L_D] = -2\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & c \\ 0 & b & \alpha & \beta \\ 0 & c & \beta & \gamma \end{pmatrix}\ , \tag{4}$$

where a, b, c, α, β and γ are six real parameters, with a, α and γ non negative. Further, these parameters must satisfy the following inequalities

$$2R \equiv \alpha + \gamma - a \geq 0\ , \qquad RS \geq b^2\ , \tag{5}$$
$$2S \equiv a + \gamma - \alpha \geq 0\ , \qquad RT \geq c^2\ , \tag{6}$$
$$2T \equiv a + \alpha - \gamma \geq 0\ , \qquad ST \geq \beta^2\ , \tag{7}$$
$$RST \geq 2\,bc\beta + R\beta^2 + Sc^2 + Tb^2\ , \tag{8}$$

which are necessary and sufficient conditions for the complete positivity of the time-evolution map ω_t generated by (1).

It should be stressed that in absence of the piece $L_D[\rho]$, pure states (*i.e.* states of the form $|\psi\rangle\langle\psi|$) are transformed by the evolution equation (1) back into pure states, even though probability is not conserved, a direct consequence of the presence of a non-hermitian part in the effective hamiltonian $H_{\rm eff}$. Only when the extra piece $L_D[\rho]$ is also present, $\rho(t)$ becomes less ordered in time due to a mixing-enhancing mechanism: it produces dissipation and possible loss of quantum coherence.[1]

A rough dimensional estimate gives the magnitude of the non-standard parameters in (4); they are at most of the order $m_K^2/M_P \simeq 10^{-19}$ GeV, with m_K the kaon mass and M_P the Planck mass;[16], [21], [10], [14] the magnitude of this estimate is not far from the

[1] Although in principle these phenomena could also contribute to the decay process of the K-mesons, their effects can be estimated to be negligible for any practical considerations;[9] the description of the K-meson "self-dynamics" by means of the effective hamiltonian (2) is therefore appropriate.

value of $\Delta\Gamma\,\epsilon_S$ and $\Delta\Gamma\,\epsilon_L$. This allows a perturbative treatment of the evolution equation in (1). It is thus possible to compare the predictions deduced from the solutions $\rho(t)$ of (1) with the available data of various observables accessible in kaon experiments.[11] Unfortunately, the accuracy of the present data, essentially from fixed target experiments, are not sufficient to draw a definite conclusion whether the constraints (5)–(8) are obeyed or not. More precise results should come from the experiments on correlated kaons in the so-called ϕ-factories.[24]

Since the ϕ-meson has spin 1, the two neutral spinless kaons produced in a ϕ-decay fly apart with opposite momenta in the meson ϕ rest-frame; their corresponding density matrix ρ_A is antisymmetric in the spatial labels. By means of the projectors onto the CP eigenstates,

$$P_1 = |K_1\rangle\langle K_1| \ , \qquad P_2 = |K_2\rangle\langle K_2| \ , \tag{9}$$

and of the off-diagonal operators,

$$P_3 = |K_1\rangle\langle K_2| \ , \qquad P_4 = |K_2\rangle\langle K_3| \ , \tag{10}$$

we can write

$$\rho_A = \frac{1}{2}(P_1 \otimes P_2 + P_2 \otimes P_1 - P_3 \otimes P_4 - P_4 \otimes P_3) \ . \tag{11}$$

The time evolution of a system of two correlated neutral K-mesons, initially described by ρ_A, can be analyzed using the sigle K-meson dynamics so-far discussed. Indeed, as in ordinary quantum mechanics, it is natural to assume that, once produced in a ϕ decay, the kaons evolve in time each according to the completely positive map ω_t generated by (1). As already remarked, this assures that the resulting total evolution map is still positive for any time and of semigroup type. We stress that this choice is the only natural possibility if one requires that after tracing over the degrees of freedom of one particle, the resulting dynamics for the remaining one be completely positive, of semigroup type and independent from the initial state of the other particle.

Within this framework, the density matrix that describes a situation in which the first K-meson has evolved up to proper time t_1 and the second up to proper time t_2 is given by:

$$\begin{aligned}&\rho_A(t_1,t_2) \equiv (\omega_{t_1} \otimes \omega_{t_2})[\rho_A]\\ &= \frac{1}{2}\Big[P_1(t_1)\otimes P_2(t_2) \ + \ P_2(t_1)\otimes P_1(t_2) \ - P_3(t_1)\otimes P_4(t_2) - P_4(t_1)\otimes P_3(t_2)\Big]\end{aligned} \tag{12}$$

where $P_i(t_1)$ and $P_i(t_2)$, $i = 1,2,3,4$, represent the evolution according to (1) of the initial operators (9), (10), up to the time t_1 and t_2, respectively. In the following, we shall set $t_1 = t_2 = t$, and simply call $\rho_A(t) \equiv \rho_A(t,t)$ the evolution of (11) up to proper time t.

The statistical description of $\rho_A(t)$ allows us to give a meaningful interpretation of the quantities

$$\mathcal{P}_{ij}(t) = \mathrm{Tr}[\rho_A(t) P_i \otimes P_j] \ , \quad i,j = 1,2 \ , \tag{13}$$

as the probabilities to have one kaon in the state $|K_i\rangle$ at proper time t, while the other is in the state $|K_j\rangle$ at the same proper time. When $i,j = 3,4$, the quantities $\mathcal{P}_{ij}(t)$ are

complex and do not represent directly joint probabilities. However, as we shall see, they can still be obtained from data accessible to experiments.

Following the discussion in [12], up to first order in all small parameters, one finds:

$$\mathcal{P}_{11}(t) = \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(1-e^{-\Delta\Gamma t}\Big)\ , \tag{14}$$

$$\mathcal{P}_{12}(t) = \frac{e^{-2\Gamma t}}{2}\ , \tag{15}$$

$$\mathcal{P}_{13}(t) = 2\, e^{-2\Gamma t}\,\frac{c+i\beta}{\Delta\Gamma_+}\Big(1-e^{-t\Delta\Gamma_+/2}\Big)\ , \tag{16}$$

$$\mathcal{P}_{22}(t) = \frac{\gamma}{\Delta\Gamma}\, e^{-2\Gamma t}\Big(e^{\Delta\Gamma t}-1\Big)\ , \tag{17}$$

$$\mathcal{P}_{23}(t) = 2\, e^{-2\Gamma t}\,\frac{c+i\beta}{\Delta\Gamma_-}\Big(1-e^{t\Delta\Gamma_-/2}\Big)\ , \tag{18}$$

$$\mathcal{P}_{33}(t) = e^{-2\Gamma t}\,\frac{2b+i(\alpha-a)}{2\Delta m}\Big(1-e^{-2it\Delta m}\Big)\ , \tag{19}$$

$$\mathcal{P}_{34}(t) = -\frac{e^{-2\Gamma t}}{2}\Big(1-2(\alpha+a-\gamma)t\Big)\ . \tag{20}$$

The remaining quantities $\mathcal{P}_{ij}(t)$ can be derived from the previous expressions by using the following properties:

$$\mathcal{P}_{ij}(t) = \mathcal{P}_{ji}(t)\ , \qquad i,j=1,2,3,4\ , \tag{21}$$

$$\mathcal{P}_{i3}(t) = \mathcal{P}^*_{i4}(t)\ , \qquad i=1,2\ , \tag{22}$$

$$\mathcal{P}_{44}(t) = \mathcal{P}^*_{33}(t)\ . \tag{23}$$

Putting $a=b=c=\alpha=\beta=\gamma=0$, one obtains the standard quantum mechanical effective description that evidentiates the singlet-like anti-correlation in $\rho_A(t)$: $\mathcal{P}_{ii}(t)\equiv 0$.

We emphasize that none of the above expressions contain the standard CP, CPT-violating parameters ϵ_S, ϵ_L. This fact makes possible, at least in principle, a direct determination of the non-standard parameters irrespectively of the values of ϵ_S, ϵ_L; one needs to fit the previous expressions of the quantities $\mathcal{P}_{ij}(t)$ with actual data from experiments at ϕ-factories.

To be more specific, we shall now explicitly show how the quantities $\mathcal{P}_{ij}$ can be directly related to frequency countings of decay events. First, notice that, given any single-kaon time-evolution $\rho\mapsto\rho(t)$, the matrix elements of the state $\rho(t)$ at time t can be measured by identifying appropriate orthogonal bases in the two-dimensional single kaon Hilbert space. The choice of the CP-eigenstates $|K_1\rangle$, $|K_2\rangle$ is rather suited to experimental tests. Indeed, since a two-pion state has the same CP eigenvalue of $|K_1\rangle$, the probability $P_t(K_1)=\langle K_1|\rho(t)|K_1\rangle$ of having a kaon state K_1 at time t is directly related to the frequency of two-pion decays at time t. Possible direct CP violating effects, the only ones allowing $K_2\to 2\pi$, can be safely neglected; they are proportional to the phenomenological parameter ε', that is expected to be very small, $|\varepsilon'|\leq 10^{-6}$.[2]

[2]The recent measures of $\mathcal{R}e(\varepsilon'/\varepsilon)$, [25],[26] seem to confirm these theoretical expectations.[27]

On the other hand, while the decay state $\pi^0\pi^0\pi^0$ has $CP = -1$, the state $\pi^+\pi^-\pi^0$ may have $CP = \pm 1$. Thus, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ to have a kaon state K_2 at time t is not as conveniently measured by counting the frequency of the three-pion decays. To avoid the difficulty, the following strangeness eigenstates can be used:

$$|K^0\rangle = \frac{|K_1\rangle + |K_2\rangle}{\sqrt{2}} , \qquad |\overline{K^0}\rangle = \frac{|K_1\rangle - |K_2\rangle}{\sqrt{2}} . \tag{24}$$

Then, the probabilities $P_t(K^0) = \langle K^0|\rho(t)|K^0\rangle$ and $P_t(\overline{K^0}) = \langle\overline{K^0}|\rho(t)|\overline{K^0}\rangle$, that the kaon state at time t be a K^0, respectively a $\overline{K^0}$, may be experimentally determined by counting the semileptonic decays $K_0 \mapsto \pi^-\ell^+\nu$, respectively $\overline{K^0} \mapsto \pi^+\ell^-\overline{\nu}$, the exchanged decays being forbidden by the $\Delta S = \Delta Q$ rule. (In the Standard Model, this selection rule is expected to be valid up to order 10^{-14}.[28]) Further, the probability $P_t(K_2) = \langle K_2|\rho(t)|K_2\rangle$ of having a kaon state K_2 at proper time t can be expressed as $P_t(K_2) = P_t(K^0) + P_t(\overline{K^0}) - P_t(K_1)$, by writing

$$|K_2\rangle\langle K_2| = |K^0\rangle\langle K^0| + |\overline{K^0}\rangle\langle\overline{K^0}| - |K_1\rangle\langle K_1| . \tag{25}$$

Hence, $P_t(K_2)$ can be measured by counting the frequencies of semileptonic decays and of decays into two pions.

In order to measure the off-diagonal elements $\langle K_1|\rho|K_2\rangle$, $\langle K_2|\rho|K_1\rangle$, we use the identity

$$|K^0\rangle\langle K^0| - |\overline{K^0}\rangle\langle\overline{K^0}| = |K_1\rangle\langle K_2| + |K_2\rangle\langle K_1| , \tag{26}$$

and extract $|K_1\rangle\langle K_2|$ from it. To do this, we need a third orthonormal basis of vectors whose projectors are measurable observables in actual experiments. An interesting possibility is based on the phenomenon of kaon-regeneration (see [29] and references therein). The idea is to insert a slab of material across the neutral kaons path; the interactions of the K^0, $\overline{K^0}$ mesons with the nuclei of the material "rotate" in a known way the initial kaon states entering the regenerator into new ones. As initial states, consider the orthogonal vectors

$$|\widetilde{K}_S\rangle = \frac{|K_1\rangle - \eta^*|K_2\rangle}{\sqrt{1+|\eta|^2}} , \qquad |\widetilde{K}_L\rangle = \frac{\eta|K_1\rangle + |K_2\rangle}{\sqrt{1+|\eta|^2}} , \tag{27}$$

where η is a complex parameter which depends on the regenerating material. By carefully choosing the material and the tickness of the slab, one can tune the modulus and phase of η in such a way to completely suppress the $\widetilde{K}_L$ component and to regenerate the $\widetilde{K}_S$ state into a K_1, just outside the material slab. Thus, the probability $P_t(\widetilde{K}_S) = \langle\widetilde{K}_S|\rho(t)|\widetilde{K}_S\rangle$ that a kaon, impinging on a slab of regenerating material in a state $\rho(t)$ at time t, be a $\widetilde{K}_S$, can be measured by counting the decays into 2π just beyond the slab. Now, the projector onto the state $|\widetilde{K}_S\rangle$ reads

$$\begin{aligned} |\widetilde{K}_S\rangle\langle\widetilde{K}_S| &= \frac{1}{1+|\eta|^2}|K_1\rangle\langle K_1| + \frac{|\eta|^2}{1+|\eta|^2}|K_2\rangle\langle K_2| \\ &- \frac{\eta}{1+|\eta|^2}|K_1\rangle\langle K_2| - \frac{\eta^*}{1+|\eta|^2}|K_2\rangle\langle K_1| . \end{aligned} \tag{28}$$

Then, from (24)–(26) and (28) it follows that

$$\begin{aligned}|K_1\rangle\langle K_2| &= f_1\,|K_1\rangle\langle K_1| \,+\, f_2\,|\widetilde{K}_S\rangle\langle\widetilde{K}_S| \\ &+ f_3\,|K^0\rangle\langle K^0| \,+\, f_4\,|\overline{K^0}\rangle\langle\overline{K^0}|\ ,\end{aligned} \tag{29}$$

where

$$f_1 = \frac{1-|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \qquad f_2 = -\frac{1+|\eta|^2}{2i\mathcal{I}m(\eta)}\ , \tag{30}$$

$$f_3 = \frac{|\eta|^2-\eta^*}{2i\mathcal{I}m(\eta)}\ , \qquad f_4 = \frac{|\eta|^2+\eta^*}{2i\mathcal{I}m(\eta)}\ . \tag{31}$$

In this way, the determination of the off-diagonal elements of $\rho(t)$ amounts to counting the frequencies of decays into two pions with or without regeneration and the frequencies of semileptonic decays:

$$\begin{aligned}\langle K_1|\rho(t)|K_2\rangle &= f_1\,P_t(K_1) + f_2\,P_t(\widetilde{K}_S) \\ &+ f_3\,P_t(K^0) + f_4\,P_t(\overline{K^0})\ .\end{aligned} \tag{32}$$

The application of these results to the case of correlated kaons is now straightforward. For sake of compactness, we identify the various kaon states with the projections Q_μ, $\mu = 1,2,3,4$, where:

$$Q_1 = |K_1\rangle\langle K_1|\ , \qquad Q_3 = |K^0\rangle\langle K^0|\ , \tag{33}$$

$$Q_2 = |\widetilde{K}_S\rangle\langle\widetilde{K}_S|\ , \qquad Q_4 = |\overline{K^0}\rangle\langle\overline{K^0}|\ . \tag{34}$$

As discussed, these operators can be measured by identifying 2π final states, in absence and presence of a regenerator (Q_1 and Q_2), and semileptonic decays (Q_3 and Q_4); the same holds for the projectors in (9) and (10), since:

$$P_1 = |K_1\rangle\langle K_1| \equiv Q_1\ , \tag{35}$$

$$P_2 = |K_2\rangle\langle K_2| = Q_3 + Q_4 - Q_1\ , \tag{36}$$

$$P_3 = |K_1\rangle\langle K_2| \equiv P_4^\dagger = \sum_{\mu=1}^{4} f_\mu\,Q_\mu\ . \tag{37}$$

Further, we denote by $P_t(Q_\mu, Q_\nu)$ the probability that, at proper time t after a ϕ-decay, the two kaons be in the states identified by Q_μ and Q_ν, respectively. Then, the determination of the quantities $\mathcal{P}_{ij}(t)$ reduces to measuring joint probabilities, *i.e.* to counting frequencies of events of certain specificied types. Indeed, one explicitly finds:

$$\mathcal{P}_{11}(t) = P_t(Q_1, Q_1)\ , \tag{38}$$

$$\mathcal{P}_{12}(t) = P_t(Q_1, Q_3) + P_t(Q_1, Q_4) - P_t(Q_1, Q_1)\ , \tag{39}$$

$$\mathcal{P}_{13}(t) = \sum_{\mu=1}^{4} f_\mu\,P_t(Q_1, Q_\mu)\ , \tag{40}$$

$$\mathcal{P}_{22}(t) = P_t(Q_1, Q_1) + P_t(Q_3, Q_3) + P_t(Q_4, Q_4) \\ \qquad + 2\Big[P_t(Q_3, Q_4) - P_t(Q_1, Q_4) - P_t(Q_1, Q_4)\Big] , \tag{41}$$

$$\mathcal{P}_{23}(t) = \sum_{\mu=1}^{4} f_\mu \Big[P_t(Q_3, Q_\mu) + P_t(Q_4, Q_\mu) - P_t(Q_1, Q_\mu)\Big] , \tag{42}$$

$$\mathcal{P}_{33}(t) = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} f_\mu\, f_\nu\, P_t(Q_\mu, Q_\nu) . \tag{43}$$

As a result of the previous analysis, the inconsistencies of models without complete positivity, besides being theoretically unsustainable, turn out to be experimentally exposable. Let P_u and P_v project onto the correlated states

$$|u\rangle = \frac{1}{\sqrt{2}}\Big(|K_1\rangle \otimes |K_1\rangle + |K_2\rangle \otimes |K_2\rangle\Big) , \tag{44}$$

$$|v\rangle = \frac{1}{\sqrt{2}}\Big(|K_1\rangle \otimes |K_2\rangle + |K_2\rangle \otimes |K_1\rangle\Big) . \tag{45}$$

The averages of these two positive observables with respect to the state $\rho_A(t)$ read

$$\begin{aligned}\mathcal{U}(t) &= \mathrm{Tr}[\rho_A(t)\, P_u] \equiv \langle u|\rho_A(t)|u\rangle \\ &= \frac{1}{2}(\mathcal{P}_{11}(t) + \mathcal{P}_{22}(t) + \mathcal{P}_{33}(t) + \mathcal{P}_{44}(t)) \qquad (46)\\ \mathcal{V}(t) &= \mathrm{Tr}[\rho_A(t)\, P_v] \equiv \langle v|\rho_A(t)|v\rangle \\ &= \frac{1}{2}(\mathcal{P}_{12}(t) + \mathcal{P}_{21}(t) + \mathcal{P}_{34}(t) + \mathcal{P}_{43}(t)) , \qquad (47)\end{aligned}$$

and, as explained before, can be directly obtained by measuring joint probabilities in experiments at ϕ-factories. On the other hand, (14)–(20) give, up to first order in the small parameters,

$$\mathcal{U}(t) = e^{-2\Gamma t}\Bigg[\frac{\gamma}{\Delta\Gamma}\sinh(t\Delta\Gamma) + \frac{b}{\Delta m}\Big(1 - \cos(2t\Delta m)\Big) \\ \qquad + \frac{a-\alpha}{2\Delta m}\sin(2t\Delta m)\Bigg] , \tag{48}$$

$$\mathcal{V}(t) = e^{-2\Gamma t}(a + \alpha - \gamma)\, t . \tag{49}$$

Thus, $\mathcal{U}(0) = \mathcal{V}(0) = 0$, whereas

$$\frac{\mathrm{d}\mathcal{U}(0)}{\mathrm{d}t} = a + \gamma - \alpha , \qquad \frac{\mathrm{d}\mathcal{V}(0)}{\mathrm{d}t} = a + \alpha - \gamma , \tag{50}$$

are both positive because of conditions (6) and (7). More in general, the mean values (46) and (47) are surely positive, for the complete positivity of the single-kaon dynamical maps ω_t implies $\rho(t) = \sum_j V_j(t)\, \rho\, V_j^\dagger(t)$,[1]–[3] where the $V_j(t)$ are 2×2 matrices such that

$\sum_j V_j^\dagger(t) V_j(t)$ is a bounded 2×2 matrix.[3] Then, the complete evolution $\rho_A \to \rho_A(t) = \sum_{i,j} [V_i(t) \otimes V_j(t)]\, \rho_A\, [V_i^\dagger(t) \otimes V_j^\dagger(t)]$ will never develop negative eigenvalues.

On the other hand, if the single-kaon dynamical map ω_t is not completely positive, inconsistencies may emerge. As an example, take the phenomenological models studied in [21], [22], where the non-standard parameters a, b, c are set to zero and $\alpha \neq \gamma$, $\alpha\gamma \geq \beta^2$. The corresponding dynamics is not completely positive: the inequalities (5)–(8) are in fact violated. In this case, one still has $\mathcal{U}(0) = \mathcal{V}(0) = 0$, but

$$\frac{\mathrm{d}\mathcal{U}(0)}{\mathrm{d}t} = \gamma - \alpha = -\frac{\mathrm{d}\mathcal{V}(0)}{\mathrm{d}t}\ . \tag{51}$$

Therefore, one of the mean values (46), (47) starts assuming negative values as soon as $t > 0$. The inconsistence is avoided only if $\alpha = \gamma$, which is a necessary condition for getting back the property of complete positivity. We stress that planned set-ups at ϕ-factories can measure the two mean values in (46) and (47) and therefore clarify also from the experimental point of view the request of complete positivity for the dynamics of neutral kaons.

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